Cellular Coverage Map as a Voronoi Diagram

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***Abstract*— The mobile cellular network coverage is normally represented by means of hexagonal topology. This structure is useful for planning frequency reuse but not appropriate for the analysis of coverage and traffic operations as handoff, paging and registration. This paper presents the service area coverage of a cellular network as an ordered order-k multiplicatively weighted Voronoi diagram. Radio parameters such as antenna height, transmission power and specific-environment propagation characteristics are used as the basis to define the proximity rule in order to generate the Voronoi diagram. The cell boundaries are the edges of the Voronoi diagram. They are defined by comparison of the radii of adjacent cells. The proximity between a mobile and a base station is determined by means of a Euclidean distance weighted by propagation parameters.**

***Index Terms*— Land mobile radio cellular systems, Path loss prediction, Voronoi diagram.**

I. Introduction

In a mobile cellular network, a mobile station (MS) is connected to the closest base station (BS) according to the system quality requirements as the received power, the signal to noise ratio (SNR) or the bit error rate (BER). These parameters are determined by transmission parameters as transmit power, antenna heights, path loss and specific-environment propagation characteristics. The cellular network solves a closest-point problem [1] when connecting a mobile to a base station.

The coverage of a cell is estimated based on the range of the radio signal of a BS. The service area coverage considers the coverage of a set of BS separated by borders. The borders between cells give to the operator important data for planning traffic operations as handoff, registration and paging. These borders represent space information to the traffic management. Specifically, in an urban environment, the cell planning engineers need geographic information from the city map. Canyon streets, hills, buildings, tunnels, arboreous zones, footbal stadium, highways, etc, compose the space information used in planning traffic operations and coverage. These elements have influence on time of processing traffic operations and thus on processing time of the Mobile Switching Center (MSC). In a highway, the mobiles move fast and it increases the handoff rate in the MSC. A financial center located in a building causes a high traffic and it may require a microcell deployment. A footbal stadium concentrates, temporarily, a large amount of people, requiring the deployment of a mobile temporary base station.

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Besides, it is necessary to know the proximity between the traffic nodes (sport stadia, financial center, etc) and base stations in order to support traffic demand with a radio signal. The coverage map is useful to combine geographic data with operation traffic planning. Geographic Information Systems (GIS) can provide these data to the cell planning engineers.

This work presents the definition of the cells borders by means of a Voronoi diagram, where the cells are the partitions, called Voronoi regions, defined by a proximity rule [2]. The cellular mobile system analyzes a number of probable MS-BS connections and chooses those with best quality. In a general way, the connection quality decreases with distance. Consequently, the connections are established on a "nearest neighbor" basis. The cell, as a set of supported users, is determined by a closest-point search and the mobiles compose the Voronoi partition.

The structure of this paper is as follows. Section 2 presents the definition and main types of the Voronoi diagrams. Section 3 presents the coverage map as a set of Voronoi diagrams. Section 4 presents a spatial analysis of the cellular network based on the coverage map. Section 5 presents an interference model and, finally, Section 6 presents the conclusions.

II. Voronoi Diagrams

The Voronoi diagram is a geometric structure that assumes the proximity (nearest neighbor) rule in associating each point in the $\mathbb{R}^n$ space to a site point closest to it. This diagram is a partition set generated by site points located inside each partition. Each partition is called a Voronoi region.

Let **x** be a point in the $\mathbb{R}^n$ space, $C = \{c_1, ..., c_N\}$ the site points set, $E(i, j)$ the edge between two site points and V_i the Voronoi region generated by c_i. The proximity relation **x** $\in V_i$ occurs according to the proximity rule

IF **x** is closer to c_i than any other site point THEN **x** $\in V_i$.

Based on this proximity rule, the Voronoi region is defined as

$$V_i = \{\mathbf{x}|D(\mathbf{x}, c_i) \leq D(\mathbf{x}, c_j), \forall j \neq i\} \quad (1)$$

where the proximity metric D is a function of distance. The equality $D(\mathbf{x}, c_i) = D(\mathbf{x}, c_j)$ defines the border between V_i and V_j. According to the definition of D, several types of Voronoi diagram can be originated. The features of the Voronoi diagrams are extensively described in [3].

A. Main types of Voronoi diagrams

This section describes the main types of Voronoi diagrams.

Generalized – The proximity metric is $D = d$ where d is the Euclidean distance. The Voronoi region is defined as

$$V_i = \{\mathbf{x}|d(\mathbf{x}, c_i) \leq d(\mathbf{x}, c_j), \forall j \neq i\}. \quad (2)$$

The edges between site points are straight lines. This is the well-known constellation diagram used to represent digital modulation schemes.

Multiplicatively weighted – The proximity metric to the multiplicatively weighted Voronoi diagram (MWVD) is $D = d/w$ where w is the weight of the corresponding site point. The Voronoi region is defined as

$$V_i = \{\mathbf{x} | \frac{d(\mathbf{x}, c_i)}{w_i} \leq \frac{d(\mathbf{x}, c_j)}{w_j}, \forall j \neq i\} \quad (3)$$

and the edges between site points are circular arcs as illustrated in Figure 1.

Directional – The proximity metric is a weighted distance whose weight is a function of the azimuth around the site point $D = d(\theta)$. The Voronoi region is defined as

$$V_i = \{\mathbf{x} | \frac{d(\mathbf{x}, c_i)}{w_i(\theta)} \leq \frac{d(\mathbf{x}, c_j)}{w_j(\theta)}, \forall j \neq i\} \quad (4)$$

where $w_i(\theta)$ is the weight in terms of the direction θ.

Power diagram – The power diagram is a space partition in which each site corresponds to a circle with center $\langle x_i, y_i \rangle$ and radius r_i [4]. The proximity metric is $D = d^2 - r^2$ and the Voronoi region is defined as

$$V_i = \{\mathbf{x} | d^2(\mathbf{x}, c_i) - r_i^2 \leq d^2(\mathbf{x}, c_j) - r_j^2, \forall j \neq i\}. \quad (5)$$

The edges between circles are straight lines. In a pair of adjacent circles, the edge approaches the center of the circle with smallest radius as shown in Figure 2.

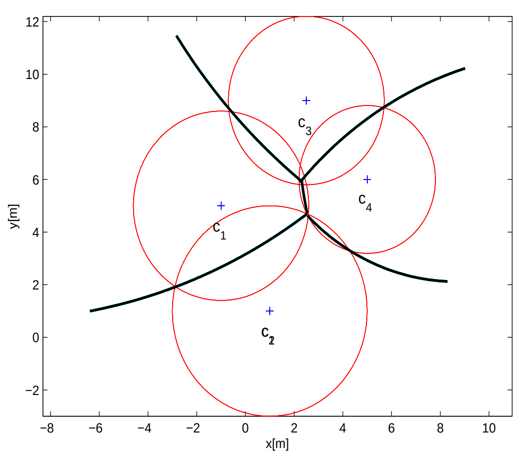

Fig. 1. Multiplicatively weighted Voronoi diagram in the plane. The edges are circular arcs.

B. The order-k Voronoi diagram

Let $\mathbf{x}$ be a point in $\mathbb{R}^n$, $C = \{c_1, ..., c_N\}$ the site points set, X a subset of C, $X \subset C$, and $\overline{X} = X - C$. The order of the Voronoi diagram k is the cardinality of the subset X: $k =| X |$. An order-k Voronoi region is closer to the site points in X than any other site point in $\overline{X}$. The order-k Voronoi region $V(X)$ is defined as [5]

$$V(X) = \{\mathbf{x} : D(\mathbf{x}, c_i) \leq D(\mathbf{x}, c_j), \forall c_i \in X, \forall c_j \in \overline{X}\}, \quad (6)$$

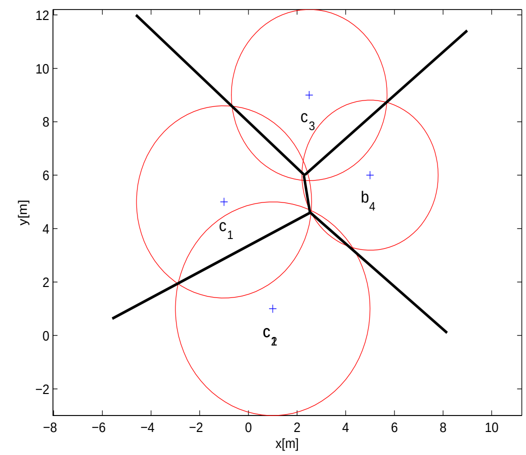

Fig. 2. Power diagram in the plane. The edges between circles are straight lines. In a pair of adjacent circles, the edge approaches the center of the circle with smallest radius.

$$\mathbb{R}^n = \bigcup_{X \subseteq C} V(X); k \in \mathbb{Z}^+; k < N.$$

The order-($k+1$) Voronoi diagram is obtained from the extensions of the edges of the order-k diagram as follows:

- Remove a site point c_i obtaining new edges. The effect of removing c_i is the elimination of the edges determined by c_i and the extension of the neighbor edges to the interior of V_i;
- Place the previously removed site point c_i back into its location and remove another site point c_j;
- Repeat for all the site points.

The result of this procedure is presented in Figure 3 as the order-1, -2 and -3 generalized Voronoi diagrams.

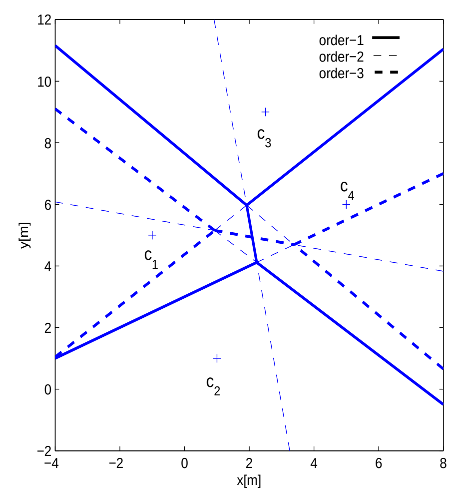

Fig. 3. Generalized orders 1, 2 and 3 Voronoi Diagrams superimposed.

C. The ordered order-k Voronoi diagram

The ordered order-k Voronoi diagram is an order-k diagram in which the proximity relations are ordered in sequence of proximity. It is obtained from the superposition of the orders $k, k-1, ..., 1$ diagrams [6]. The resulting spatial tesselation has the following features: *i)* Each Voronoi region represents the domain of a set of site points; *ii)* Proximity relations are ordered in sequence of proximity. A region denoted as $O(i, j, p)$ is the locus of all the points in $\mathbb{R}^n$ closest to the site point c_i and farthest to the site point c_p. The site points in O are ordered in a sequence of proximity.

D. The ordered order-k multiplicatively weighted Voronoi diagrams

As defined in section II-C, the ordered order-k Voronoi diagram gives the proximity relations between site points in a sequence of proximity. Now, the multiplicative weighting is combined to yield the ordered order-k multiplicatively weighted Voronoi diagram, in which, a Voronoi region, depicted as $O(X), X = \{c_i, c_j, c_p, ..., c_w\}$, is the locus of all the points closest to, firstly, the site point c_i, second to the site point c_j such that c_w is the farthest from $O(X)$. Each edge $E^{(\xi)}(i, j)$ divides the plane into half-planes $\pi_i^{(\xi)}, \pi_j^{(\xi)}$, where ξ indicates the order of the edge. The Voronoi region is the intersection of the half-planes

$$O(X) = \bigcap_{n \in X, \xi \in [1,k]} \pi_n^{(\xi)}, \tag{7}$$

For instance, see the region

$$O(3, 2, 1) = \pi_3^{(1)} \cap \pi_2^{(2)} \cap \pi_1^{(3)}$$

illustrated in Figure 4.

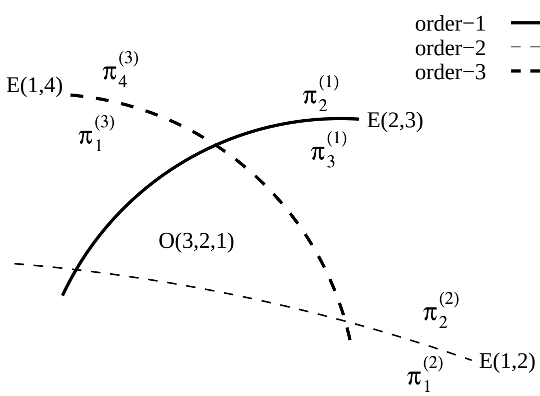

Fig. 4. Limiting edges of a multiplicatively weighted ordered order-3 Voronoi region in the plane.

III. The Service Area as a Voronoi Diagram

The coverage can be defined by the received radio signal at the mobile. The downlink field strength becomes more attenuated far from the BS. Thus, there is a minimum accepted received power that activates the receiver. When this limit is achieved, the current connection is broken and a handoff or outage operation is executed. In order to estimate the cell limits, it is necessary to predict the path loss. Several models aim at predicting this loss through analytic and measurement based methods. The appropriate, environment-specific, model must be chosen for accurate path loss prediction. For macrocells, the most common models are: Lee, Okumura-Hata and COST-Hata. For microcells, the COST-Walfish-Ikegami, the Xia-Bertoni model and ray-tracing method are useful [7]. The predicted path loss depends on frequency, antenna heights, distance and propagation environment characteristics. Generically, the path loss can be expressed, in dB, as [8]

$$L = a + b\log(d) \tag{8}$$

where a and b are parameters dependent on the model and the distance d is given in Km.

A. Cell radius

Assume an environment having the same path loss in all the directions around the BS. With a transmit omnidirectional antenna, the mean boundary of the cell is a circumference whose radius depends on the propagation parameters. The received power equation

$$P_r = P_t + G_t + G_r - L \tag{9}$$

is used to estimate the cell radius, where P_r is given in dBm, P_t is the BS transmit power in dBm and G_t and G_r are the BS and MS antenna gain in dB, respectively. Substituting (8) into (9), the received power P_r is obtained in terms of the propagation parameters a and b. Let the received power threshold be denoted as Z, thus the condition $P_r \geq Z$ defines the cell. At the cell border $P_r = Z$ and the distance d corresponds to the cell radius given in Km

$$r = 10^{\left(\frac{P_t + G_t + G_r - a - Z}{b}\right)}. \tag{10}$$

There is also a statistical method described in [9], [10] to estimate the cell radius. According to that method, the signal envelope is considered to follow the Rayleigh, Lognormal and Suzuki distributions.

B. The two-cell model

Consider two adjacent cells as seen in Figure 5 and a mobile connected to BS_1. When the mobile is far from BS_1 and approaches BS_2, the system analyzes the current, and the alternative, connection quality. If the current connection quality degenerates below a certain threshold, the system changes the connection to BS_2. A proximity rule can be defined based on the received power

IF $P_{r1} \geq P_{r2}$, the mobile is closer to BS_1.
ELSE, the mobile is closer to BS_2,

where P_{rj} is the downlink received power transmitted by the jth BS. The locus of the condition $P_{r1} = P_{r2}$ is a circumference (shown in Figure 5 as a thick line) which represents the border of the two cells. It is illustrated also in three dimensions in Figure 6, where the transmit power decreases according to the Okumura-Hata model.

The border between two cells is the locus of the condition

$$\frac{d_1}{w_1} = \frac{d_2}{w_2} \tag{11}$$

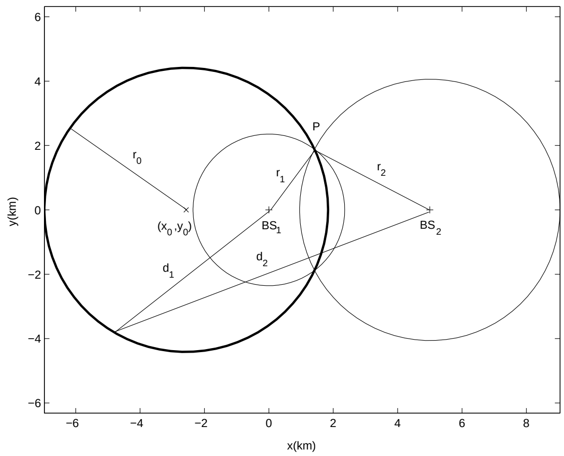

Fig. 5. The two-cell model. The border between cells is the locus of the condition $P_{r1} = P_{r2}$ which is shown as a thick line.

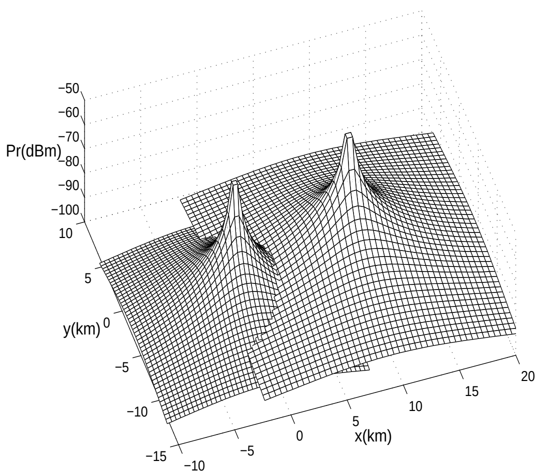

Fig. 6. The two-cell model in a three-dimension diagram. The locus of the border condition $P_{r1} = P_{r2}$ can be seen as a circular arc between the cells.

where d_i/w_i is the distance weigthed by w_i; d_i is the distance between iBS and the border of the cell which is given in terms of the BS location coordinates

$$d_i = \sqrt{(x - x_i)^2 + (y - y_i)^2} \tag{12}$$

and the equation (11) yields

$$(x-x_1)^2 + (y-y_1)^2 = \left(\frac{w_1}{w_2}\right)^2 [(x-x_2)^2 + (y-y_2)^2] \tag{13}$$

This is the equation of a circumference whose center $\langle x_0, y_0 \rangle$ and radius r_0 are given as

$$x_0 = \frac{(w_{12})^2 x_2 - x_1}{(w_{12})^2 - 1}; \tag{14}$$

$$y_0 = \frac{(w_{12})^2 y_2 - y_1}{(w_{12})^2 - 1}; \tag{15}$$

$$r_0 = \left| \frac{d_{12} w_{12}}{(w_{12})^2 - 1} \right| \tag{16}$$

where d_{12} is the distance separating the BSs and $w_{12} = w_1/w_2$ correponds to the distance ratio d_1/d_2. The circumferences with radii r_1, r_2 intersect at point P. It gives

$$\frac{d_1}{d_2} = \frac{r_1}{r_2}. \tag{17}$$

Expressing (17) as

$$\frac{d_1}{r_1} = \frac{d_2}{r_2}, \tag{18}$$

yields the definition of the proximity rule in an MWVD, expressed in (3). Let the BSs be represented by site points in an MWVD, thus the weights, according to (11), correspond to the cells radii

$$w_i = r_i \tag{19}$$

and the distance ratio is given by the cells radii ratio

$$w_{ij} = \frac{r_i}{r_j}. \tag{20}$$

The borders of the cells can be represented by an MWVD [11], since the link BS-MS is established according to a proximity rule expressed as a function of the received power. The base station works as a site point determining a Voronoi region by means of its radio signal. The border between two adjacent cells is, as in an MWVD, a circular arc [12]. For the particular case where $w_i = w_j$ ($r_i = r_j$), the border is a straight line (a circular arc with infinite radius). The distance ratio w_{ij} can be obtained for the whole service area by taking cells pairwise. The distance ratio represents the neighboring relationship of the BSs. For two adjacent BSs, the proximity rule is

IF $d_i \leq w_{ij} d_j$ THEN the mobile is closer to BS$_i$.
ELSE the mobile is closer to BS$_j$.

Figure 7 shows the edges between two site points in terms of the distance ratio.

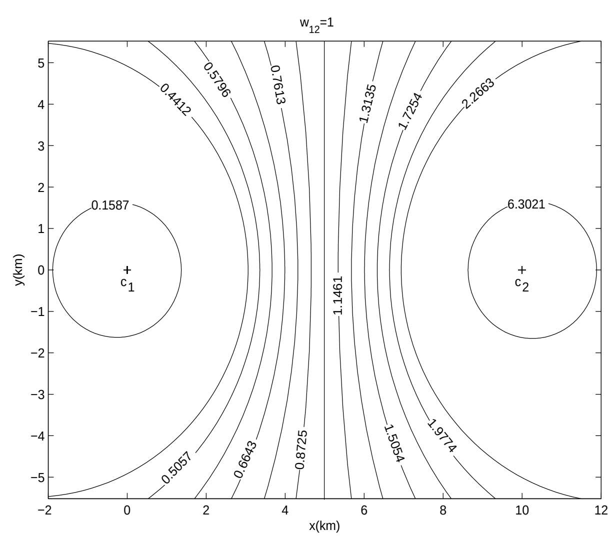

Fig. 7. Family of edges of two adjacent site points in terms of the distance ratio.

C. Sectored cells

For a sectored cell, the transmit power is a function of the antenna pattern. If the antenna gain is assumed constant within the sector, the cell sector can be represented by a sector of a circle whose radius is determined by (10). This case is shown in Figure 8, where the cell of BS_1 is omnidirectional and those of BS_2 and BS_3 are three-sectored. The neighboring relationship is analyzed by taking adjacent cells pairwise. Consider the border of $BS_2(s_3)$ and $BS_3(s_1)$. The proximity rule defined in (3) applied to the border of sectored cells gives the following distance ratio

$$w_{i(s_p)j(s_q)} = \frac{r_i(s_p)}{r_j(s_q)}, \tag{21}$$

where r is the radius of the p-th sector s of the i-th BS.

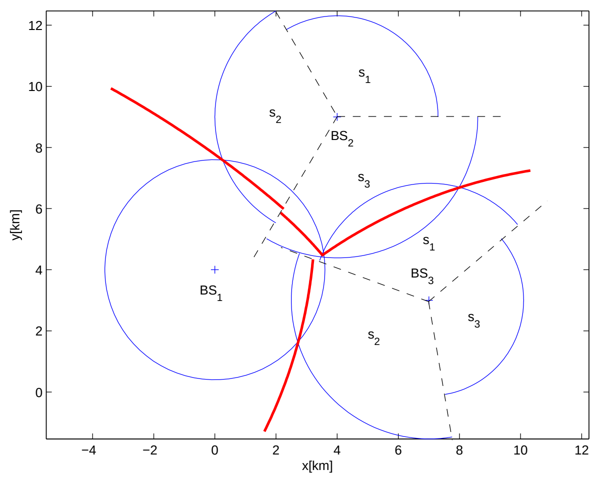

Fig. 8. Sectored cells represented by a Voronoi diagram. A distance ratio is obtained for each border between sectors. The borders are shown as thick lines.

D. Hierarchical cells

A microcell is usually used for hot traffic spots or small coverage holes in a hierarchical structure. A microcell can be deployed by using a low transmit power, a low antenna height or a tilted antenna. This is the case of overlaid coverage of two cells when a microcell is inside an umbrella cell. Geometrically, this structure can be represented by an MWVD [13] as shown in Figure 9, where the MWVD principle stated in (3) is applied to represent overlaid coverage. As seen in Figure 9, the border between umbrella and microcell is a circumference. It is coherent to the radio propagation theory. The radio signal of the umbrella BS is stronger than the radio signal of the microcell in the area surrounding the periphery of the microcell.

E. The coverage map

The input data to plot the coverage map is: transmit power and location of BSs, propagation environment characteristics, received power threshold and cells radii. To estimate the cells radii, assume the Okumura-Hata model for path loss

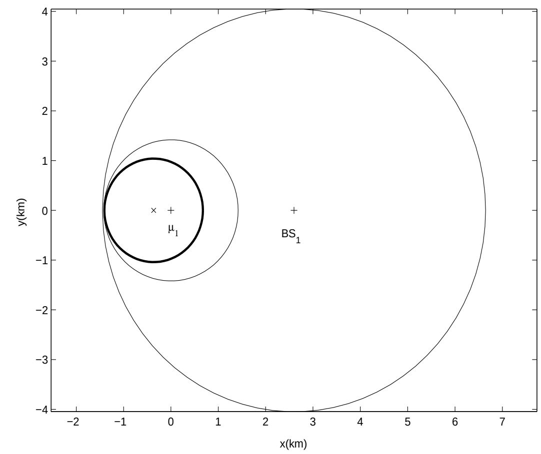

Fig. 9. Umbrella/microcell representation using the MWVD. The microcell is denoted μ_1. The border umbrella/microcell is shown as a thick line.

prediction in a flat terrain, medium sized city with medium trees density, BS antenna heights in the range of 30 to 200 m, above rooftop, mobile antenna height between 1 and 10 m, a 150-1000 MHz frequency range and a link maximum lenght of 20 km. Other assumed data is: the antenna gains sum $G_t + G_r = 10$ dB, the mobile antenna height is 3 m, the carrier frequency is 850 MHz and the received power threshold is -90 dBm.

F. Distance ratio

The cell radius is obtained from (10). The distance ratio is obtained using (20). The parameters a and b in (8) are obtained from the path loss prediction formula. For the environment previously described, the Okumura-Hata formula is

$$L{=}69.55 + 26.16\log(f) - 13.82\log(h_b) - a(h_m) + (44.9 - 6.55\log(h_b))\log(d), \tag{22}$$

$$a(h_m) = (1.1\log(f) - 0.7)h_m - (1.56\log(f) - 0.8). \tag{23}$$

The antenna height is denoted h_b for BS and h_m for MS, f is the carrier frequency. In order to obtain the path loss in the form $L = a + b\log(d)$, the parameters a and b are obtained directly from (22)

$$a = 69.55 + 26.16\log(f) - 13.82\log(h_b) - a(h_m), \tag{24}$$

$$b = 44.9 - 6.55\log(h_b). \tag{25}$$

The BS data is shown in Table I and the distance ratio in Table II. The cells are represented by the MWVD shown in Figure 10 in which the circumferences radii represent the weights of the BSs and do not compose the diagram. The Voronoi region contours are circular arcs whose radii are related to the radii of the neighbor cells according to (16) and (20).

BS	Location (km)	Power (dBm)	Antenna height (m)	Cell radius (km)	Okumura-Hata parameters	
1	(2,10)	37	55.0	3.605	a_1=118.34	b_1=33.50
2	(5,15)	32	65.0	2.779	a_2=117.33	b_2=33.02
3	(7,3)	40	61.0	4.687	a_3=117.72	b_3=33.20
4	(7,9)	40	56.0	4.474	a_4=118.23	b_4=33.44
5 s_1	(11,14)	37	38.2	3.000	a_5s_1=120.52	b_5s_1=34.53
5 s_2	(11,14)	37	60.0	3.774	a_5s_2=117.81	b_5s_2=33.25
5 s_3	(11,14)	40	45.3	4.000	a_5s_3=119.49	b_5s_3=34.05
6	(12,8)	35	55.0	3.142	a_6=118.34	b_6=33.50
$\mu 1$	(4.5,1)	28	46.6	1.800	$a_{\mu 1}$=119.31	$b_{\mu 1}$=33.97

TABLE I

BS DATA: LOCATION, POWER AND ANTENNA HEIGHTS; OKUMURA-HATA PARAMETERS. THE CELL OF BS_5 IS THREE-SECTORED: s_1, s_2, s_3. THE BS μ_1 IS A MICROCELL.

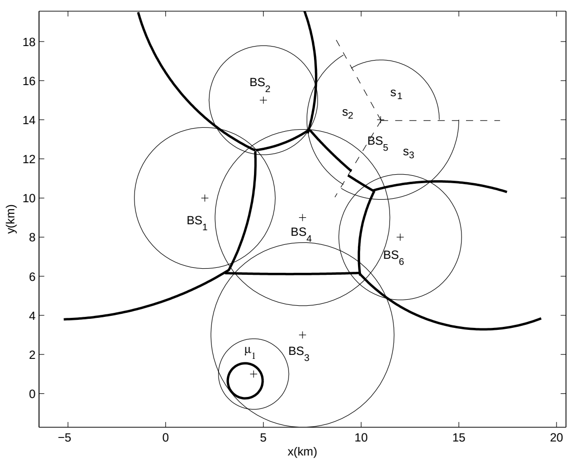

Fig. 10. A cluster of six BS. The Voronoi diagram is composed of circular arcs shown as thick lines. The cell of BS_5 is three-sectored. The cell of BS_3 has a microcell denoted as μ_1.

w_{12}	w_{13}	w_{14}	w_{24}	$w_{25(s_2)}$	w_{34}
1.2973	0.7691	0.8058	0.6212	0.7364	1.0477

w_{36}	$w_{45(s_2)}$	$w_{45(s_3)}$	w_{46}	$w_{5(s_3)6}$	$w_{3(\mu 1)}$
1.4918	1.1856	1.1185	1.4239	1.2010	2.6042

TABLE II

THE DISTANCE RATIO BETWEEN TWO ADJACENT CELLS.

IV. SPATIAL ANALYSIS OF THE COVERAGE

Some traffic operations including handoff, paging, registration and outage are closely related to the proximity between BSs. These proximity relations can also have influence in cochannel interference, frequency reuse and channel allocation scheme. Spatial information as terrain morfology, buildings, roads, tunnels, etc, can be derived from GIS and proximity between cells can be derived from the coverage map as a Voronoi diagram. These information can be used in spatial traffic models. For example, a user in a tunnel can have the call temporarily dropped and the reconnection becomes a frequent operation in the nearest BS serving that area. A highway, next to the border of adjacent cells, makes the handoff rate augmented, requiring a modification in the handoff limiting levels, turning faster the handoff execution, in order to avoid unsuccessful handoffs. The cell planning engineering can plan coverage and handoff based on specific spatial data as, for example, a shadow area among dense region of high buildings. In order to illustrate, a cluster of four cells is analyzed. Its characteristics are:

- The COST-Hata model is used for path loss prediction;
- Carrier frequency: 1800 MHz;
- Omnidirectional antennas;
- Antenna gains: $G_t + G_r = 9$ dB;
- Received power threshold: -100 dBm;
- Urban environment.

Further transmission data is given in Table III. The corresponding order-3 diagram is shown in Figure 11. The Voronoi edges have three descriptors: center, radius and distance ratio as shown in Table IV. The following spatial analysis is presented.

- Shopping and financial centers, highways, airports, etc, acquired from GIS, can be seen as nodes of demand[1]. The proximity between a node of demand and a BS is valuable for spatial traffic modeling and planning;
- The farthest BS can be identified for planning frequency reuse and channel allocation schemes. For instance, an order-4 diagram shows regions of the type $O(i, j, p, q)$. It means that BS_i is the nearest station and BS_q is the farthest one; BS_i can support primarily the traffic in O and BS_q can borrow or reuse channels of BS_i.
- The Voronoi regions identify locations proximity: $O(i, j, p)$ is an area covered by BSs i, j, p, in this sequence. A handoff may occur primarily between BS_i and BS_j and, secondly between BS_i and BS_p. If O has large dimensions, slow handoffs are expected among BSs i, j, p. Else, if O has small dimensions, fast handoffs are expected;
- Information about spatial traffic can be explored. For instance, if a highway extends from $O(3, 4, 2)$ towards $O(2, 4, 3)$, a high handoff rate between BSs 2 and 3 can be expected. The time to process the handoff is expected to be small because of the high velocity of the mobiles in the highway;
- All the regions denoted $O(i, ...)$ are closer to BS_i than other BS. This identifies the coverage of BS_i. It is

[1]Zones of the cell in which the traffic is considered to be uniform and constant in a certain time interval.

observed in Figure 11 that the intersection

$$\bigcap_{j,p,...\neq i} O(i,j,p,...) \tag{26}$$

is not a circle, but a polygon;

- The traffic can be planned based on spatial information. For example, BS_3 and BS_4 can support part of the BS_2 traffic in a certain time interval for originating calls from $O(2,3,4)$ and $O(2,4,3)$. Further, BS_1 can support the originating calls from $O(2,1,3)$ and $O(2,3,1)$ when BS_2 is heavy loaded;
- In a handoff prioritized scheme, a handoff in progress must be concluded. For this purpose, a number of radio resources (channel, time slot) are reserved to perform handoff. If a call is originated to a heavy loaded BS, the system can be planned to transfer this call to the closest BS. Related to the region $O(i,j,p)$, the heavy loaded BS is BS_i and BS_j can support the originating calls to BS_i;

The technique named *cell breathing* is exclusive of the CDMA systems. In order to aliviate a heavy loaded cell, the BS reduces its transmit power. This action reduces the coverage area and some of the users are transferred to neighbor cells by handoff. Since this technique has influence on the coverage area, a spatial analysis can be made as follows:

- Assume BS_i is heavy loaded. When it breathes, its overlapping area changes dinamically and the proximity relations in the neighborhood is altered. For each step in power decreasing of BS_i, all the neighbor edges move in a nonlinear manner. These changes affect the load of the neighbor cells and the handoff operations. The transmit power reduction alters the edges according to Equations (10), (14), (15) and (16), and the new edge $E^*(i,j)$ is defined in terms of the distance ratio

$$w_{ij} = 10^{\frac{P^*_{ti}+G_{ti}+G_r-a_i-Z}{b_i} - \frac{P_{tj}+G_{tj}+G_r-a_j-Z}{b_j}} \tag{27}$$

where $P^*_{ti} = P_{ti} - \Delta P$ is the power of BS_i after a step of power variation ΔP [14]. An example is shown in Figure 12 where BS_2 is breathing. The edges $E(2,j)$ move to $E^*(2,j)$ for a reduction of 5% in r_2.

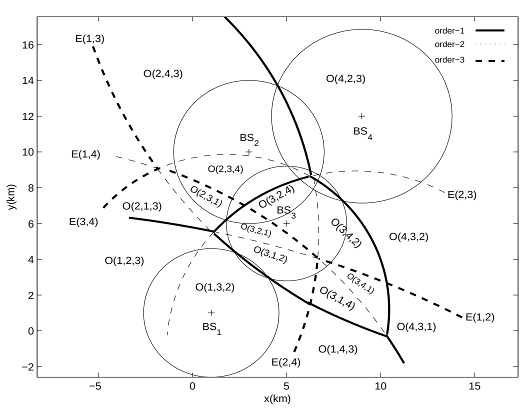

Fig. 11. Ordered order-3 multiplicatively weighted Voronoi diagram, representing four adjacent BSs. Orders 1, 2 and 3 diagrams are superimposed.

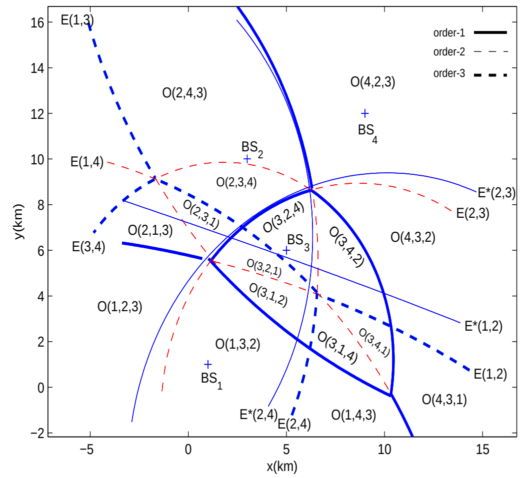

Fig. 12. BS_2 are breathing. The Voronoi edges move and the Voronoi regions change their dimensions in a nonlinear manner.

V. Interference and Outage Contour

The outage is a condition in which a mobile user is completely deprived of service by the system, a service condition below a threshold of acceptable performance [15]. This situation is caused by cochannel interference plus noise. The outage is a probabilistic phenomenon, because the interference occurs randomly, when the channel allocation system fails and allocates the same channel to adjacent cells. The outage occurs when the signal to interference plus noise ratio (SINR) falls below a predetermined protection ratio. The interference phenomenon can be geometrically approached because the radio signal suffers attenuation with distance. The distance dependence of the radio signal makes it possible to determine a mean boundary around a station in which the cochannel interference may occur.

A simplified interference model is presented in Figure 13. Two adjacent base stations BS_1 and BS_2 are shown, assuming the BS_2 as the interfering source. The interference may occur in the two following ways:
1. From a base station onto a mobile (downlink);
2. From a mobile onto a base station (uplink).
This model considers only the situation in item 1, the mobile station as an interfering source is not considered. Let d_1 be the distance BS_1-MS_1 and d_2 the interfering link distance BS_2-MS_1. According to the Apollonius theorem, the locus of the distance ratio d_1/d_2 is a circumference [16] whose center and radius are given by (14), (15) and (16). The outage condition can be analyzed by the SINR formula

$$\mathrm{SINR}_1 = 10\log\left(\frac{p_{r1}}{p_{r2}+n_0}\right) \tag{28}$$

where SINR_1, in dB, refers to the interfered downlink, p_{r1} is the received power of the target signal, p_{r2} is the received power of the interfering signal and n_0 is the Additive White Gaussian Noise (AWGN) in watts. According to the expression (28)

IF SINR $\geq \lambda_{th}$ the mobile is free of outage.

BS	Location (km)	Power (dBm)	Antenna height (m)	Cell radius (km)	COST-Hata parameters
1	(1,1)	40	41.6	3.6	a_1=129.81 b_1=34.29
2	(3,10)	40	50.9	4.0	a_2=128.70 b_2=33.72
3	(5,6)	34	73.5	3.2	a_3=126.49 b_3=32.67
4	(9,12)	43	48.9	4.8	a_4=128.94 b_4=33.83

TABLE III
BS DATA AND PATH LOSS PARAMETERS.

	E(1,2)	E(1,3)	E(1,4)	E(2,3)	E(2,4)	E(3,4)
Center (km)	(-7.5,-37.6)	(20,24.8)	(-9.3,-13.1)	(8.6,-1.1)	(-10.6,5.5)	(1.8,1.2)
Radius (km)	43.869	27.131	23.318	9.962	17.205	8.655
Distance ratio	0.900	1.125	0.750	1.250	0.834	0.667

TABLE IV
VORONOI EDGES AND DISTANCE RATIO.

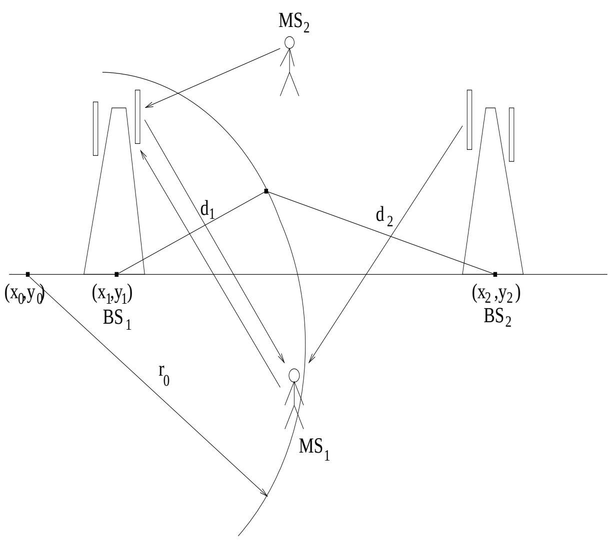

Fig. 13. Interference model in an arbitrary cellular network. The downlink signal of BS_2 is interfering on a mobile MS_1. The uplink signal of MS_2 is interfering on BS_1.

ELSE the mobile is subject to outage,

where λ_{th} is the protection ratio given in dB. It is reasonable to consider $p_{r2} >> n_0$. Therefore, the SINR_1 can be treated as SIR_1 and given by

$$\text{SIR}_1 = P_{t1} - L_1 - (P_{t2} - L_2), \tag{29}$$

where P_{t1} is the BS_1 transmit power and P_{t2} is the interfering transmit power in dBm. For a given protection ratio λ_{th}, the distance ratio

$$w_{12} = \frac{d_1}{d_2} \tag{30}$$

can be used to define the outage contour which corresponds to the circumference of the Apollonius circle, i.e., the locus of the ratio d_1/d_2. This contour is a bisector dividing the plane into half-planes, each one representing the coverage of the BSs. It is also an edge of an MWVD. All the points surrounding BS_1, where the distance ratio is verified, determine the locus of the condition $\text{SIR} = \lambda_{th}$ and define the outage contour. The input data to plot the MWVD are: *(1)* BS locations $\langle x_i, y_i \rangle$ and *(2)* the distance ratio w_{12}. From

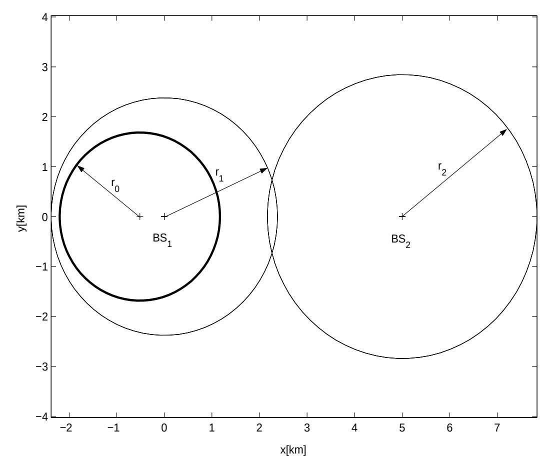

Fig. 14. The outage contour, shown as thick line, is a circumference. It is also an edge of a multiplicatively weighted Voronoi diagram.

the Equation (29), the expression

$$\begin{aligned}\lambda_{th} = P_{t1} + G_{t1} + G_{r1} - a_1 - b_1 \log(d_1) - P_{t2} \\ -G_{t2} - G_{r2} + a_2 + b_2 \log(d_2).\end{aligned} \tag{31}$$

is derived. From (31), the distance ratio d_1/d_2 can be numerically computed according to the algorithm described as follows. Consider a pair of BS: BS_1 and BS_2. A point $\mathbf{x}$ between them is taken. This point moves along the line joining the BSs determining two distances: $d_1 = \overline{\text{BS}_1, \mathbf{x}}$ and $d_2 = \overline{\text{BS}_2, \mathbf{x}}$. The received power from each BS is computed in terms of the distance. For each iteration, d_1 and d_2 are stored. When the condition $P_{r1} - P_{r2} > \lambda_{th}$ fails, it means that the border has been found $P_{r1} - P_{r2} \approx \lambda_{th}$, and the values d_1, d_2 are used to calculate $w_{12} = d_1/d_2$.

INPUT:	BS antenna height (h_b) and location $\langle x, y\rangle$, transmit power(P_t), protection ratio (λ_{th}).
OUTPUT:	d_1, d_2, w_{12}.
Step 1.	Initialize $d_1 = 0, d_2 = d_{12}$.
Comment:	*d_{12} is the distance separating two BSs.*
Step 2.	Compute the parameters of the path loss prediction model: a, b.
Step 3.	Compute P_{r1}, P_{r2}.
Step 4.	IF $P_{r1} - P_{r2} > \lambda_{th}, d_1^{++}, d_2^{--}$, GO TO 3. ELSE $w_{12} = d_1/d_2$.
Step 5.	EXIT.

Consider the transmission data shown in Table V. The carrier frequency is 1800 MHz, COST-Hata path loss prediction model, received power threshold is -100 dBm, mobile antenna height is 3 m. The outage contour is the circumference described by center: $\langle -0.5, 0\rangle$ km and radius: $r_0 = 1.671$ km, shown in Figure 14. The method to obtain the distance ratio is graphically shown in Figure 15.

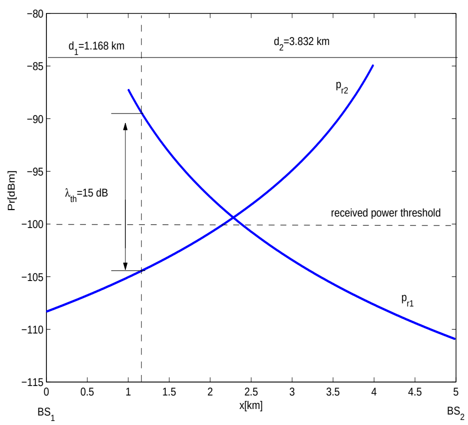

Fig. 15. Method to compute the distance ratio. The point where $P_{r1} - P_{r2} = \lambda_{th}$ defines d_1, d_2 and $w_{12} = d_1/d_2$.

VI. Conclusions

The coverage map of a cellular network is used to plan coverage and traffic operations. This map can be superimposed to the map of the city, combining geographic data with radio coverage. Some traffic operations including handoff, registration, paging and outage are closely related to the proximity between BSs. These proximity relations can also have influence in cochannel interference, frequency reuse and channel allocation scheme. Spatial information as terrain morfology, buildings, roads, tunnels, footbal stadium, etc, can be derived from GIS, and proximity between cells can be derived from the Voronoi diagrams. The spatial traffic can be modeled by proximity relations between cells. Physical characteristics of the area covered are taken as part of the cell. The cells representation using Voronoi diagrams is applicable to omni and sectored cells, as well as to the hierarchical cells structure. The base station is represented as a site point and the distance is weighted by the cell radius. The order-k diagrams show the common coverage of a set of cells. These intersection regions are closely related to the handoff occurrence. The proximity of a set of BSs allows to plan overlapping coverage and shared traffic. The interference between cells are represented by outage contours. The estimation of the cell radius is not an exact method but involves a statistical error depending on the path loss prediction model. Therefore, the predicted borders of the cells incorporate this error.

References

[1] M. I. Shamos and D. Hoey, “Closest-point problems,” in *Proc. 16th IEEE Annual Symposium on Foundations of Computer Science*, Berkeley, US, 1975, pp. 151 – 162.

[2] J. Basch, L. Guibas, and L. Zhang, “Proximity problems on moving points,” in *Proceedings 13th Annual ACM Symposium on Computational Geometry*, Nice, FR, June 4–6, 1997, pp. 344 – 351.

[3] F. Aurenhammer, “Voronoi diagrams - A survey of a fundamental geometric data structure,” *ACM Computing Surveys*, vol. 23, pp. 345 – 405, Sept. 1991.

[4] F. Aurenhammer, “Power diagrams: properties, algorithms and applications,” *SIAM Journal on Computing*, vol. 16, pp. 78 – 96, Feb. 1987.

[5] D.-T. Lee, “On k-nearest neighbor Voronoi diagrams in the plane,” *IEEE Transactions on Computers*, vol. C-31, n. 6, pp. 478 – 487, Jun. 1982.

[6] M. Held and R. B. Williamson, “Creating electrical distribution boundaries using computational geometry,” *IEEE Transactions on Power Systems*, vol. 19, issue 3, pp. 1342 – 1347, Aug. 2004.

[7] COST-231 European comission, “Digital mobile radio towards future generation systems,” Final report, Eraldo Damosso (ed.), chapter 4, 1999.

[8] M. Hata, “Propagation loss prediction models for land mobile communications,” *Proceedings of the International Conference on Microwave and Millimeter Wave Technology*, ICMMT, Beijing, CH, 1998, pp. 15 – 18.

[9] M. D. Yacoub, “Cell Design Principles. In: Jerry D. Gibson. (Org.). The Communications Handbook.” 2 ed. Boca Raton: CRC Press, 2002, vol. 1, pp. 1 – 13.

[10] E. J. Leonardo, “Métodos estatísticos para a determinação da área de cobertura de células e micro-células em sistemas de rádio móvel.” [in portuguese], Master Thesis, State University of Campinas, UNICAMP, São Paulo, BR, 1992.

[11] J. N. Portela and M. S. Alencar, “Outage contours using a Voronoi diagram,” *Proceedings of the Wireless Communication and Networking Conference*, WCNC, Atlanta, US, 2004, pp. 2383 – 2386.

[12] M. D. Yacoub, “*Foundations of Mobile Radio Engineering*”, CRC Press, Boca Raton, 1993.

[13] J. N. Portela and M. S. Alencar, “Spatial analysis of the overlapping cell area using Voronoi diagrams,” *Proceedings of the International Microwave and Optoelectronics Conference*, IMOC 2005, Brasília, BR, 2005.

[14] A. Jalali, “On cell breathing in CDMA networks,” *IEEE International Conference on Communications*, ICC’98, Atlanta, US, 1998, pp. 985 – 988.

[15] B.C. Jones and D. J. Skellern, “Outage contours and cell size distributions in cellular and microcellular networks,” *Vehicular Technology Conference, IEEE 45th*, 1995, Chicago, US, pp. 145 – 149.

[16] H. Haruki and T. M. Rassias, “A new characteristic of Möbius transformations by use of Apollonius points of triangles,” *Journal of Mathematical Analysis and Applications*, vol. 197, n. 1, pp. 14 – 22, Jan. 1996.

BS	Location (km)	Power (dBm)	Antenna height (m)	Cell radius (km)	COST-Hata parameters
1	(0,0)	43	45	2.356	a_1=130.34 b_1=34.01
2	(5,0)	45	50	2.806	a_2=129.77 b_2=33.77

TABLE V

BS DATA TO OBTAIN THE OUTAGE CONTOUR.

José do Nascimento Portela was born in Ceará, Brazil, in 1956. He works in technological education since 1984 in Centro Federal de Educação Tecnológica, Telecommunications Department. He received his Bachelor Degree in Electrical Engineering from the Federal University of Ceará (UFCE), Brazil, 1982, his Master Degree in Electrical Engineering, from Federal University of Paraíba (UFPB), Brazil, 1992, and Doctor degree in Federal University of Campina Grande, UFCG, 2006, Department of Electrical Engineering. He is member of the Brazilian Telecommunications Society (SBrT) in which he has been acting as article reviewer. His research interest includes Cellular Mobile Communication Networks, Channel Modeling, Computational Geometry and Software for Education.

Marcelo Sampaio de Alencar Marcelo Sampaio de Alencar was born in Serrita, Brazil in 1957. He received his Bachelor Degree in Electrical Engineering, from Universidade Federal de Pernambuco (UFPE), Brazil, 1980, his Master Degree in Electrical Engineering, from Universidade Federal da Paraiba (UFPB), Brazil, 1988 and his Ph.D. from University of Waterloo, Department of Electrical and Computer Engineering, Canada, 1993. Marcelo S. Alencar has more than 25 years of engineering experience and he is currently IEEE Senior Member. Since 1995, he is Chair Professor at the Department of Electrical Engineering, Federal University of Campina Grande, Brazil. He worked, between 1982 and 1984, for the State University of Santa Catarina (UDESC). He is founder and President of the Institute for Advanced Studies in Communications (IECOM). He has been awarded several scholarships and grants from CNPq and IEEE Foundation, an achievement award for contributions to the Brazilian Telecommunications Society (SBrT), an award from the Medicine College of the Federal University of Campina Grande (UFCG) and an achievement award from the College of Engineering of the Federal University of Pernambuco. He published over 170 engineering and scientific papers, three chapters and six books. He supervised three post-doctoral fellows, five Ph.D. theses and 16 Master's dissertations. Marcelo S. Alencar has contributed in different capacities to the following scientific journals: Editor of the Journal of the Brazilian Telecommunication Society; Member of the International Editorial Board of the Journal of Communications Software and Systems (JCOMSS), published by the Croatian Communication and Information Society (CCIS); Member of the Editorial Board of the Journal of Networks (JNW), published by Academy Publisher; Editor-in-Chief of the Journal of Communication and Information Systems (JCIS). He is member of the SBrT-Brasport Editorial Board and has been involved as a volunteer with several IEEE and SBrT activities. He is a Registered Professional Engineer. He has been acting as reviewer for several scientific journals. He is a columnist for the traditional Brazilian newspaper Jornal do Commercio, since April, 2000. Marcelo S. Alencar is Vice-President External Relations of the SBrT. He is member of the Institute of Electronics, Information and Communication Engineering (Japan), member of SBMO (Brazilian Microwave and Optoelectronics Society), member of SBPC (Brazilian Society for the Advancement of Science) and member of SBEB (Brazilian Society for Biomedical Engineering).